**ANSWER OF QUESTIONS BANK DISCRETE MATH**

**PROPOSITIOAL LOGIC, PREDICATE & QUANTIFIER**

1. Let *W*(*r*) mean room *r* is painted white. Let *I*(*r*,*b*) mean that room *r* is in building *b*. Let *L*(*b*,*u*) mean that building *b* is on the campus of US university *u*. The statement is *u**b**L*(*b*,*u*) *r**I* (*r*,*b*)*W*(*r*).
2. We need to negate each part and swap “and” with “or”
3. Kate will not take a job in industry and will not go to graduate school.
4. John doesn’t know Java or doesn’t know Calculus.
5. James is not young or he is not strong.
6. Rica will not move to Oregon and will not move to Washington.
7. *T* (*p* *q*)((*q* *p*)) Original statement

*T* (*p* *q*) ∨ ((*q* *p*)) Definition of implication (twice)

*T* (*p* ∨ *q*) ∨ (*q* *p*) DeMorgan’s law (twice)

*T* (*p* ∨ *q*) ∨ (*q* ∨ *p*) DeMorgan’s law (again)

*T* (*p* ∨ *q*) ∨ (*p*∨*q*) DeMorgan’s law (again)

*T* *T* Complement law

Note that the last step is because anything or’ed with its complement (here it was (*p*∨*q*) or’ed with its complement) is a tautology.



|  |  |
| --- | --- |
| ( *p* ∧ ( *p* ∧ *q*)) ∧ (￢*p* ∨ *q*) ≡ ( *p* ∧ *q*) | Original statement |
| ( *p* ∧ *p* ∧ *q*) ∧ (￢*p* ∨ *q*) ≡ ( *p* ∧ *q*) | Associativity of AND (removing of parenthesis) |
| ( *p* ∧ *q*) ∧ (￢*p* ∨ *q*) ≡ ( *p* ∧ *q*) | Idempotent law |
| (( *p* ∧ *q*) ∧ ￢*p*)∨ (( *p* ∧ *q*) ∧ *q*) ≡ ( *p* ∧ *q*) | Distributive law |
| (*q* ∧ *p* ∧ ￢*p*) ∨ (*p* ∧ *q* ∧ *q*) ≡ ( *p* ∧ *q*) | Associativity of AND (removing of parenthesis) |
| (*q* ∧ *F*) ￢ (*p* ∧ *q* ∧ *q*) ≡ ( *p* ∧ *q*) | Negation law |
| *F* ∨ (*p* ∧ *q* ∧ *q*) ≡ ( *p* ∧ *q*) | Domination law |
| (*p* ∧ *q* ∧ *q*) ≡ ( *p* ∧ *q*) | Identity law |
| (*p* ∧ *q*) ≡ ( *p* ∧ *q*) | Idempotent law |

1. Let *P*(*x*) be “*x* is perfect”; let *F*(*x*) be “*x* is your friend”; and let the domain be all people.
2. *x* (*P*(*x*) equivalent with *x* *P*(*x*)
3. *x* (*F*(*x*) *P*(*x*))
4. *x*(*F*(*x*)*P*(*x*)) equivalent with (*x F*(*x*))(*x P*(*x*))
5. *x F*(*x*)) ∨ (*x* *P*(*x*))
6. Let *M*(*x*, *y*) be “*x* has sent *y* an email message” and *T*(*x*, *y*) be “*x* has telephoned *y*”, where the domain consists of all students in your class.
7. *M*(Deborah, Jose)
8. *x* *y* (*x* ≠ *y*  *M*(*x*, *y*)  *M*(*y*, x)))
9. *x M*(*x*, x)
10. *x* *y* (*x* ≠ *y* *M*(*x*, *y*)  *T*(*y*, x)))

**RULES OF INFERENCE**

1. Answer:
2. ￢*p* First hypothesis
3. ￢*q* ∨ *p* Second hypothesis
4. ￢*q* Disjunctive syllogism on steps 1 and 2
5. ￢*r* ∨ *q* Third hypothesis
6. ￢*r* Disjunctive syllogism on steps 3 and 4
7. Answer:

|  |  |  |
| --- | --- | --- |
| 1. | *x* ¬ *P*(*x*) | Premise |
| 2. | ¬ *P*(*c*) | Existensial instantiation from (1) |
| 3 | *x* (*P*(*x*) ∨ *Q*(*x*)) | Premise |
| 4. | (*P*(*c*) ∨ *Q*(*c*)) | Universal instantiation from (3) |
| 5. | *Q*(*c*) | Disjunctive syllogism from (4) and (2) |
| 6. | *x* (¬ *Q*(*x*) ∨ *S*(*x*)) | Premise |
| 7. | ¬ *Q*(*c*) ∨ *S*(*c*) | Universal instantiation from (6) |
| 8. | *S*(*c*) | Disjunctive syllogism from (5) and (7) |
| 9. | *x* (*R*(*x*) → ¬ *S*(*x*)) | Premise |
| 10. | *R*(*c*) → ¬ *S*(*c*) | Universal instantiation from (9) |
| 11. | ¬ *R*(*c*) | Modus tollens from (8) and (10) |
| 12. | *x* ¬ *R*(x) | Existensial generalization from (11) |

1. Answer:

*x*(*P*(*x*)*S*(*x*)) *P(x)* means *x* is a hummingbird

*x*(*Q*(*x*) *R*(*x*)) *Q*(*x*) means *x* is large

*x*(*R*(*x*)*S*(*x*)) *R*(*x*) means *x* lives on honey

*x*(*P*(*x*)*Q*(*x*)) *S*(*x*) means *x* is richly colored

The universe of discourse is all birds

*x*(*P*(*x*) *S*(*x*)) *P(x)* means *x* is a hummingbird

*x*(*Q*(*x*) ∨ *R*(*x*)) *Q*(*x*) means *x* is large

*x*(*R*(*x*) *S*(*x*)) *R*(*x*) means *x* lives on honey

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The universe of discourse is all birds

*p* *s*

*q* *r*

*r* *s*

*p* *q*

1. *p* *s* 1st hypothesis
2. *r* *s* 3rd hypothesis
3. *s* *r* Contrapositive of step 2
4. *p* *r* Hypothetical syllogism of steps 1 and 3
5. *q* *r* 2nd hypothesis
6. *r* *q* Commutative law from step 5
7. *r* *q* Definition of implication from step 6
8. *p* *q* Hypothetical syllogism of steps 4 and 7
9. Let *p* = “The house is next to a lake”

Let *q* = “The treasure is in the kitchen”

Let *r* = “The tree in the front yard is an elm”

Let *s* = “The tree in the back yard is an oak”

Let *t* = “The treasure is in the garage”

Let *u* = “The treasure is buried under the flagpole”

Our clues are:

* 1. *p*  *q*
  2. *¬p* *u*  *r* *¬s*
  3. *t* *¬s*
  4. *¬u*  *¬r*
  5. *¬q*

Our steps with the rules of inference are:

* 1. *p* *q* 1st hypothesis
  2. *q* 5th hypothesis
  3. *p* Modus tollens on steps 1 and 2
  4. *p* *u* Addition on step 3
  5. *p* *u* *r* *s* 2nd hypothesis
  6. *r* s Modus ponens on steps 4 and 5
  7. *r* Simplification on step 6
  8. *u**r* 4th hypothesis
  9. *u* Modus tonens on steps 8 and 9

So, the treasure is buried under the flagpole.

1. *r* = Dominic goes to the racetrack

*h* = Helen gets mad

*p* = Ralph plays cards all night

*c* = Carmela gets mad

*v* = Veronica is notified

Premises:

* 1. *r* → *h*
  2. *p* → *c*
  3. *h* ∨ *c* → *v*
  4. ￢*v*

Conclusion: ￢*r* ∧ ￢*p*

* 1. *h* ∨ *c* →*v* 3rd hypothesis
  2. ￢*v* 4th hypothesis
  3. ￢(*h* ∨ *c*) Modus tollens on steps 1 and 2
  4. ￢*h* ∧ ￢*c* DeMorgan’s law on step 3
  5. ￢*h* Simplification of step 4
  6. *r* → *h* 1st hypothesis
  7. ￢*r* Modus tollens on steps 5 and 6
  8. ￢*c* Simplification of step 4
  9. *p* → *c* 2nd hypothesis
  10. ￢*p* Modus tollens on steps 8 and 9
  11. ￢*r* ∧ ￢*p* Conjunction on steps 7 and 10

**PROOF**

1. Note: for question 13b you only need to choose one method of proof.
2. Given two numbers *m* and *n*, if *m* is even and *n* is odd, then *m+n* is odd
3. **Indirect proof**

We want to prove the contrapositive: **If *m+n* is even, then *m* is odd OR *n* is even**. Note that it doesn’t matter which of *m* or *n* is odd – just that either one of them is odd or one of them is even. The only way an *or* statement can be false is if both are false – namely that *m* is even and *n* is odd are both false. However, since we don’t care which of *m* or *n* is odd and even, we can swap them and fulfill the or statement.

Assume that *m*+*n* is even, then *m*+*n* = 2*k*

Suppose that *m* is odd and *n* is odd, then *m*=*2k*+1 for some integer *k*, and *n*=2*l*+1 for some integer *l*

Then *m*+*n* =2*k*+1+2*l*+1=2*k*+2*l*+2=2(*k*+*l*+1), so it is even.

Note: you can also proof it by swap them, namely *m* is even and *n* is even

The other possibility is that *m* is odd and *n* is even which result in false hypothesis. But remember that an implication with false hypothesis is always true.

**Proof by contradiction**

Assume that the implication is false, namely that the antecedent is true (namely, that *m* is even and *n* is odd), and that the consequence is false (namely, that *m*+*n* is even). Let *m* = 2*k* , and let *n* = 2*l* +1, for some integers *k* and *l*

*m* + *n* = 2*k* + 2*l* + 1

*=* 2(*k + l*)+1

As *m+n* is 2 times an integer plus one (that integer is *k+l*), *m+n* must be odd. This contradicts our assumption that *m+n* is even.

1. Answer
2. Proof of an existential by example: 3, 5, 7. (Rosen 3.1.25, page 224.)
3. Proof by counter example: 22-1 = 3, which is not composite. (Rosen 3.1.22, page 224.)
4. Indirect proof: show that if *n* is not prime, then the sum of its divisors is not *n*+1. If *n* is not prime, then it at least has factors 1, *k*, and *n*, where 1 < *k* < *n*, the sum of which is greater than *n*+1. (Rosen 3.1.38, page 224.)
5. **Direct proof**

The statement translates into *p*→*q*, where *p* means *m* is even, and *q* means *m*+7 is odd. For the direct proof, we assume *p* is true, and show that *q* must always be true. If *m* is even, then *m*=2*k*, where *k* is some integer (this is the definition of even numbers). Thus, *m*+7 = 2*k*+7 = 2(*k*+3)+1, where *k*+3 is an integer (as *k* was an integer). This is the definition of odd numbers (two times an integer plus one). Thus, *m*+7 must be odd.

**Indirect proof**

The statement translates into *p*→*q*, where *p* means *m* is even, and *q* means *m*+7 is odd. For the indirect proof, we prove the contrapositive. The contrapositive is ￢*q*→￢*p*, which translate to “if *m*+7 is even, then *m* is odd”. If *m*+7 is even, then *m*+7 = 2*k*+1, where *k* is some integer (definition of odd numbers). Solving for *m*, we get *m* = 2*k*-6 = 2(*k*-3), where *k*-3 is an integer (as *k* was an integer). This is the definition of even numbers (two times another integer). Thus, *m* must be even.

**Proof by contradiction**

The statement translates into *p*→*q*, where *p* means *m* is even, and *q* means *m*+7 is odd. For the proof by contradiction, we assume that the statement is false, and arrive at a contradiction. For the statement to be false, we assume that *p* must be true, and *q* must be false. Thus, we assume that *m* is even, and *m*+7 is even. If *m* is even, then *m*=2*k*, where *k* is some integer (this is the definition of even numbers). Thus, *m*+7 = 2*k*+7 = 2(*k*+3)+1. This, however, is the definition of odd numbers, and therefore *m*+7 must be odd. This contradicts our assumption that *m*+7 is even. Thus, the statement must be true, as the one case where it is false (i.e. when *m* is even, and *m*+7 is even) cannot occur.

1. *n* is even 🡪 7*n* + 4 is even

Via direct proof:

Suppose *n* is even, *n* = 2*k*

7*n* + 4 = 7(2*k*) + 4 = 14*k* + 4 = 2(7*k* + 2)

Since 7*n* + 4 = 2(7*k* + 2), is 2 times an integer, thus it is even

7*n* + 4 is even 🡪 *n* is even

Its contraposition: *n* is odd 🡪 7*n* + 4 is odd

Via indirect proof:

Suppose *n* is odd, *n* = 2*k* + 1

7*n* + 4 = 7(2*k* + 1) + 4 = 14*k* + 11 = 2(7*k* + 5) + 1

Since 7*n* + 4 = 2(7*k* + 5) + 1, is 2 times another integer plus 1, thus it is odd.

Therefore, we can conclude that the statement “*n* is even if and only if 7*n* + 4 is even” is true.

1. We will show that the four statements are equivalent by showing that

a 🡪 b, b 🡪 c, c 🡪 d, and d 🡪 a

1. First we prove a 🡪 b

Suppose *n* is even, *n* = 2*k* for some integer *k*

Then *n* + 1 = 2*k* + 1 so it is odd. This shows that a) implies b)

1. Next, b 🡪 c

Suppose that *n* + 1 is odd, *n* + 1 = 2*k* + 1 for some integer *k*

3*n* + 1 = 2*n* + (*n* +1) = 2*n* + 2*k* + 1 = 2(*n*+*k*) + 1, which shows that 3*n* + 1 is odd. Thus, b) implies c)

1. c 🡪 d

Suppose that 3*n* + 1 is odd, so 3*n* + 1 = 2*k* + 1 for some integer *k*

3*n* = (2*k* + 1) – 1; 3*n* = 2*k*. So, 3*n* is even.

This shows that c) implies d)

1. d 🡪 a

its contraposition: if *n* is odd, then 3*n* is odd

suppose that *n* is odd, *n* = 2*k* + 1

3*n* = 3(2*k* + 1) = 6*k* + 3 = 2(3*k* + 1) + 1, so 3*n* is odd. It shows that c) implies d)

These, complete our proofs that the four statements are equivalent.

1. |*a* – *c*| = |*b* – *c*| is equivalent to the disjunction of two equations,

*a* – *c* = *b* – *c* **or** *a* – *c* = -*b* + *c*

The first of these, *a* – *c* = *b* – *c* is equivalent to *a* = *b*, which contradict the assumption made in the question (that is *a* ≠ *b*).

So, the original equation is equivalent to *a* – *c* = -*b* + *c*.

*Alternatively, you may think that |a – c| = |b – c| is also equivalent to*

*-a + c = b – c* ***or*** *-a + c = -b + c.*

*Similarly, you will find that -a + c = -b + c is equivalent to a = b, which contradict the assumption made in the question (that is a ≠ b).*

*So, the original equation is equivalent to –a + c = b – c which is equivalet to*

*a – c = -b + c.*

By adding b + c to both sides and dividing by 2, we see that this equation is equivalent to c = (a+b)/2. Thus there is a unique solution.

Furthermore, this c is an integer because a and b are odd integers. The sum of two odd integers is even integer.

**SETS**

1. We can spesify a set by:
2. List all of the elements, e.g. *A* = {1, 2, 3, 4, 5}
3. Use ellipsis when general pattern of the elements is obvious, e.g. *B* = {0, 1, 2, 3, …}
4. Use set-builder notation, e.g. *D* = {*x* | *x* is prime and *x* > 2}
5. Answer:
6. {-1, 1} 🡪 kalau ada yg jawab betulkan saja
7. {0, 1, 4, 9, 16, 25, 36, 49, 64, 81} 🡪 kalau ada yg jawabannya kurang 0 betulkan saja
8. {} or ∅ 🡪 kalau jawabnya {∅} salah
9. Answer:
10. Yes
11. No
12. No
13. They aren’t equivalent since the elements of A x B x C consist of 3 tuples (a, b, c) where a ∈ A, b ∈ B, and c ∈ C; whereas the elements of (A x B) x C look like ((a, b), c) – ordered pairs, the first coordinate is again an ordered pair.
14. 10
15. Answer:
16. *B* ⊆ *A* or *B* = ∅
17. Nothing, because this is always true (Commutative law)
18. A ∩ B *=* ∅ 🡪 Alternative: (1) A and B are disjoint (saling lepas); (2) B is empty set
19. B = ∅
20. Let A, B, and C be sets. Show that . (Specify the law you used in every steps).

